ELASTOVISCOUS LIQUID FLOW DEVELOPMENT IN A TUBE UNDER CONSTANT PRESSURE HEAD

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The nonisothermal transient process of flow of a nonlinearly elastoviscous liquid in a circular tube is numerically studied for the case of pulsed application of a pressure head.

Polymer solutions and mixtures as well as emulsion and suspension type liquid compositions widely used in many technological processes possess elastic properties. These properties manifest themselves in transient deformation regimes. We shall consider a case of great practical importance, the development of flow in a tube under the action of a constant pressure head. The experimental studies of Meissner [1] show that in flowing media with elastic properties flow development differs from the monotonic velocity field development in nonelastic liquids. This problem was considered in a number of theoretical studies [2-6] for an elastoviscous liquid with constant properties [2-4], for the nonlinear-viscous Oldroyd model with one relaxation time [5, 6], and also for nonelastic liquids [7, 8]. But until the present there has been no theoretical study of the effect upon flow development of factors such as existence of a relaxation time spectrum, which is intrinsic to all real polymers, nonisothermal conditions, and the character of the dependence of relaxation time and relaxation moduli on deformation rate.

The present study is a numerical investigation of nonstationary nonisothermal flows of nonlinear-elastoviscous liquids in long coaxial tubes with annular cross section ($R_1 \leqslant r \leqslant R_2$). For a tube with length significantly greater than the size of the hydrodynamic and thermal input sections, the mathematical formulation of the problem may be written as

$$\rho \frac{\partial v_z}{\partial t} = -\frac{\partial p}{\partial z}(t) + \frac{1}{r}\frac{\partial}{\partial r}(rT_{rz}), \quad v_z|_{r=R_1} = v_z|_{r=R_2} = 0. \tag{1}$$

The outer and inner cylinders are maintained at constant temperatures θ_2 and θ_1, and dissipative heat production may be neglected for liquids with moderate viscosity. In this case, across the flow area we have the well-known logarithmic temperature distribution $(\theta - \theta_1)/(\theta_2 - \theta_1) = (\ln r/R_1)/(\ln R_2/R_1)$.

To describe the liquid behavior we use a nonlinear integral rheological equation of state with the memory function dependent on the invariants of the deformation rate tensor. Real polymer liquids can be considered incompressible. The general form of such an equation for the excess stress tensor then appears as [9]:

$$\mathbf{T} = \int_{-\infty}^{t} m[t-t', S_D(t')]\left[\left(1+\frac{\varepsilon}{2}\right)(\mathbf{C}_t^{-1}(t') - \mathbf{E}) + \frac{\varepsilon}{2}(\mathbf{C}_t(t') - \mathbf{E})\right]dt', \tag{2}$$

$$m = \sum_{k=1}^{\infty} \frac{\eta_k}{\lambda_k^2} f_k(S_D(t')) \exp\left[-\int_{t'}^{t} \frac{g_k(S_D(t''))}{\lambda_k}\, dt''\right],$$

where $S_D^2 = 2\mathrm{tr}\mathbf{D}^2$; $\mathbf{D}$ is the deformation rate tensor; $\mathbf{E}$ is a unit tensor; ε is a model parameter; $\mathbf{C}_t$, $\mathbf{C}_t^{-1}$ are Cauchy and Finger finite deformation tensors describing the geometric non-

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linearity. The physical nonlinearity of Eq. (2) is described by the dependence of the memory function on the deformation rate.

Differentiating Eq. (2) with respect to time, we obtain an equivalent system of differential equations [10]:

$$T_{rz} = \sum_{k=1}^{\infty} T_{k,rz}; \quad \frac{\partial T_{k,rz}}{\partial t} + \frac{g_k}{\lambda_k} T_{k,rz} = p_k \frac{\partial v_z}{\partial r}; \quad \frac{\partial p_k}{\partial t} + \frac{g_k}{\lambda_k} p_k = \frac{\eta_k}{\lambda_k^2} f_k, \tag{3}$$

which together with Eq. (1) describe the fluid flow. In shear flow the nonlinearity of the model of Eq. (2) is determined solely by the dependence of f_k, g_k on S_D: as $S_D \to 0$ f_k, $g_k \to 1$. The relaxation time spectrum in the linear viscoelasticity range is approximated by an exponential [9] $\lambda_k = \lambda/k^{\alpha}$; $\eta_k = \eta_0/\zeta(\alpha)k^{\alpha}$. Here α is the spectrum characteristic, $1.5 \leqslant \alpha \leqslant 8$; $\zeta(\alpha)$ is the Rieman zeta function; λ is the greatest relaxation time in the spectrum; and η_0 is the initial Newtonian viscosity. With appropriate choice of the functions f_k, g_k many models well known in the literature may be obtained from Eq. (2) [9]. The following liquid types were considered: with constant properties $f_k = g_k = 1$, nonlinear models of Bird–Carro (BC) $f_k = 1/(1 + \lambda_k^2 S_D^2)$, $g_k = 1$, Meister (M) $f_k = 1$, $g_k = 1 + \frac{c}{\sqrt{2}} \lambda_k S_D$, and Macdonald–Bird–Carro (MBC) $f_k = (1 + \lambda_k' S_D)/(1 + \lambda_k S_D)$; $g_k = (1 + \lambda_k' S_D)^{3/2}/(1 + \lambda_k S_D)^{1/2}$. The value $\lambda' = 0.2\lambda$ was used in the calculations. This choice of models permits analysis of various factors: In the BC model change in shear modulus is considered; in M, changes in relaxation time; while in the MBC model, changes in both shear moduli and relaxation times are considered.

In view of the strong temperature dependence of rheological properties of polymer solutions, the flow character is determined to a great extent by the temperature field. For nonisothermal flows in Eq. (2) we employ the temperature–time superposition principle [11], according to which, neglecting the temperature–density correction $\eta = \eta_s a(\theta)$; $\lambda = \lambda_s a(\theta)$, and $a(\theta) = \exp\left\{\frac{Q}{R}\left(\frac{1}{\theta} - \frac{1}{\theta_s}\right)\right\}$. The subscript s refers to some reference temperature, for which we choose the mean temperature between the cylinders. The direction of the temperature head is characterized by the parameter $\nu = (\theta_1 - \theta_2)/\theta$; at $\nu < 0$ the outer cylinder is heated, and at $\nu > 0$, the inner. The parameter ν was varied over the range 0.1-0.5. The temperature dependence of the viscosity is described by the parameter $b = R\theta_s/Q$, with b varied from 0.05 to 0.1. The character of the flow is defined by the parameters: α, $El = \lambda\eta/\rho h^2$, the elasticity number, equal to the ratio of the relaxation times and the flow development time in the viscous liquid gap; $We = \lambda V/h$, the Weissenberg number, defined as the ratio of the relaxation times to the characteristic shear velocity (here $V = \left(\frac{\partial p}{\partial z}\right) h^2/\eta$), and $\delta = \frac{h}{R_1} \equiv \frac{R_2 - R_1}{R_1}$, which defines the relative curvature of the gap. For $El \ll 1$ the elastic properties of the liquid have practically no effect and the liquid behaves as a nonlinear viscous one with viscosity $\eta(\dot{\gamma}) = \sum_{k=1}^{\infty} \eta_k f_k(\dot{\gamma})/g_k^2(\dot{\gamma})$, while as $We \to 0$ $f_k \to 1$, $g_k \to 1$, and system (3) describes a liquid with constant properties. The values of α, El, and We were varied over the following ranges: α, 2-6; El, 0.1-100; We, 1-25, which ranges correspond to real materials. Thus, for example, for an 0.01% aqueous solution of polyacrylamide at h = 10 cm, $El \approx 0.4$, $\alpha \approx 2.5$. The ratio of El and We was selected so that the Reynolds number $Re = We/El$ corresponded to the laminar range.

We will consider nonstationary flows of a liquid with constant properties $f_k = g_k = 1$. In the initial moments the velocity increases linearly at every point of the gap $v_z \approx -\frac{1}{\rho}\frac{\partial p}{\partial z} t$, and tangent stresses are absent before arrival of the shear wave. Waves moving from the channel walls distort the planar velocity profile and tangent stresses appear. Increases in both

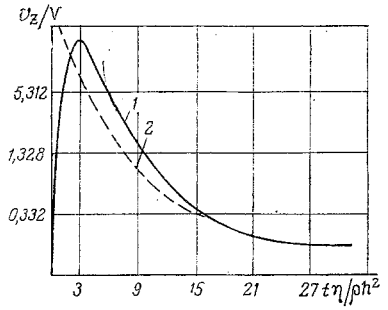

Fig. 1

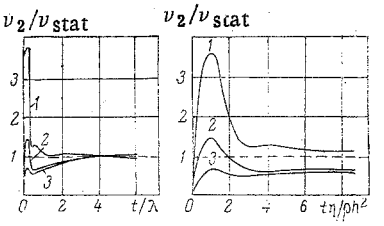

Fig. 2

Fig. 1. Velocity values at maximum of stationary profile for elastoviscous liquid with constant properties at El = 100, $\alpha = 4$, $\delta = 0.1$, k = 2 for numerical (1) and quasistationary (2) solution.

Fig. 2. Velocity values at maximum of stationary profile for MBC model at El = 10, $\alpha = 2$, $\delta = 0.1$, We = 1 (1), 10 (2), 20 (3). Wave stage shown with expanded $t\eta/\rho h^2$ scale.

the elasticity number El and the spectral parameter α cause wave propagation to be retarded. A characteristic feature of elastoviscous liquid behavior is the presence of two stages: a wave stage with rapidly changing stress profiles, and a quasistationary stage in which a stationary stress distribution is established and the liquid inertia is insignificant. In this second stage the liquid motion is determined solely by creep at constant stress. The duration of the wave stage is essentially determined by inertial properties ($t \sim \rho h^2/\eta$) while the duration of the second stage is determined by the relaxation time ($t \sim \lambda$). These two stages are clearly separated at $El \gtrsim 100$. In the quasistationary stage, when liquid inertia may be neglected, Eq. (1) can be written as

$$\frac{\partial p}{\partial z} 1(t) = \frac{1}{r} \frac{\partial}{\partial r} \left[r \int_0^t \Psi(t-t') \frac{\partial v_z(r, t')}{\partial r} dt' \right],$$

where $\Psi(t-t') = \sum_{k=1}^{\infty} \frac{\eta_k}{\lambda_k} \exp\left(\frac{t-t'}{\lambda_k}\right)$ is the relaxation function. Using a Laplace transform, we obtain

$$v_z = \frac{1}{4} \frac{dJ}{dt} \frac{dp}{dz} \left[r^2 - R_1^2 - \frac{R_1^2 - R_2^2}{\ln R_2/R_1} \ln \frac{r}{R_1} \right], \tag{4}$$

where the creep function J is related to the relaxation function by the expression [11] $\int_0^t J(t-\tau)\Psi'(\tau)\,d\tau = 1$. In order to compare the numerical results with the analytical solution, we use the Oldroyd model (which can be obtained from Eq. (3) at k = 2), for which

$$\Psi = \frac{\eta}{\lambda_1} [(1-\beta)\exp(-t/\lambda_1) + \beta\lambda_1 1(t)], \quad \beta = \lambda_2/\lambda_1,$$

$$J(t) = t/\eta + \lambda_1(1-\beta)[1 - \exp(-t/\beta\lambda_1)]/\eta.$$

Figure 1 shows curves of the change in flow velocity at the maximum point as obtained numerically for two relaxation times, and from Eq. (4). Beginning at time $t\eta/\rho h^2 \sim 15$ these values practically coincide.

For an inelastic liquid the quasistationary stage is absent and the exit to stationary flow is determined solely by inertial properties. Velocity and stress development is monotonic. In an elastoviscous liquid the velocity and stress oscillate about their stationary values. For example, the increase in tangent stress on the wall above its stationary value reaches 20% for El = 10, $\alpha = 2$, $\delta = 0.1$. Increase in elasticity number El and the parameter

α leads to manifestation of elastic properties — the amplitude and number of oscillations increase: at El = 10, α = 2, δ = 0.1 the maximum velocity value for the first oscillation $v_{max}/V = 0.51$, at El = 30, α = 2 $v_{max}/V = 0.72$, and at El = 10, α = 3 $v_{max}/V = 0.81$. At $\alpha < 3$ (for El $\simeq$ 10) the velocity values oscillate above the stationary ones, approaching them from above, while for $\alpha \geqslant 3$ the oscillation minima lie below the stationary values. Stationary flow of an elastoviscous liquid with constant properties coincides with the flow of a Newtonian liquid with the same initial viscosity. With increasing curvature of the annular gap v_{max} shifts toward the inner cylinder. The case $\delta = 0$ corresponds to a plane channel, with v_{max} located on the axis. For $\delta \leqslant 9$ the deviation from the axis is relatively small. For $\delta = 0.1$ it is practically absent, while at $\delta = 1$ it is 6%, and at $\delta = 10$, 18%. This agrees with the results of [12]. For $\delta \geqslant 10$ as flow develops the velocity maximum in the initial stage moves from the channel axis to the inner wall. The duration of this displacement $t\eta/\rho h^2 \sim 1$.

We will consider the flow of a nonlinear-elastoviscous liquid for the BC, M, and MBC models. In these models it is also possible to distinguish two stages in flow development — wave and quasistationary. As in the case of a liquid with constant properties, increase in α and El increases the duration of the initial stage and the passage of shear waves. The shear wave propagates more slowly with increase in pressure gradient (parameter We). The duration of the first stage is $t \sim \rho h^2/\eta$, while for the second, $t \sim \lambda$.

Similarly to the case of the liquid with constant properties, the velocity and stress profiles oscillate upon exit to the stationary state, and increase in relaxation time and α leads to more intense manifestation of elastic properties, the amplitude and number of oscillations increasing. Thus, change in the elasticity number from 10 to 100 increases the value of v_{max}/v_{stat} at the peak of the first oscillation by 1.9 times for the BC model, 1.35 times for MBC, and 1.85 times for M. In the BC and MBC models at α = 2, after one oscillation the velocity exits monotonically to the stationary value, while in the M model at α = 2, We = 4 there is an increase in the amplitude of the second oscillation, which disappears with increase in We. This can be explained by the fact that with increase in We the relaxation times decrease, and for high shear velocities the oscillatory character is less clearly expressed. For the BC and MBC models the effect of change in We is weaker. With increase in pressure head the effective viscosity falls, so that the stationary value of the relative velocity increases. For the BC model at We = 1 and 10, $v_{stat}/V = 0.136$ and 0.913; for the MBC model at We = 1 and 10, $v_{stat}/V = 0.137$ and 0.376. Increase in Weissenberg number leads to suppression of elastic effects, the oscillation peak decreasing. At sufficiently high We (We $\simeq$ 10 for BC, We $\simeq$ 4 for M, We $\simeq$ 20 for MBC) the velocity value oscillations lie below the stationary values, and the exit to the stationary state is more similar to the nonlinear viscous case (Fig. 2).

Analysis of the exit to the stationary state shows that two types of velocity change are possible: The value approaches the stationary value either from above (at low We) or from below, passing through a minimum. In the quasistationary stage the dependence of velocity on time t/λ for identical α and We but differing El is practically the same, since there is no dependence on density.

For nonisothermal flow in a stationary temperature field the velocity and stress distributions change due to temperature dependence of the rheological characteristics. The viscosity near the heated wall decreases and the velocity profile shifts toward that wall. This shift is larger, the larger the temperature head. In a planar gap the velocity profile shift is symmetric relative to the isothermal case upon change in the direction of the temperature head. With increase in curvature the velocity maximum becomes less than the isothermal value if the inner cylinder is heated ($\nu > 0$), or greater than the isothermal value if the outer cylinder is heated ($\nu < 0$). As is evident from Fig. 3, the shear wave which occurs in the initial stage propagates more rapidly at the less heated wall. The tangent stresses at the heated wall are less than the isothermal value, but are greater at the cold wall. Thus, for the MBC model at El = We = 10, δ = 1, α = 2, $T_1/T_{01} = 0.647$, $T_2/T_{02} = 1.203$ if $\nu = 0.25$, and $T_1/T_{01} = 1.630$, $T_2/T_{02} = 0.665$ if $\nu = -0.25$. Here T_{01} and T_{02} are the tangent stress for isothermal flow, and T_1 and T_2 are the same quantity for nonisothermal flow.

The results obtained coincide qualitatively with the experiments of [1], in which flow development in low density polyethylene was studied. Upon exit to the stationary state the liquid performed two oscillations. The experimental data agree best with the calculations performed for a liquid with constant properties at α = 2, El = 10, and for the nonlinear models with We $\simeq$ 1 and El $\simeq$ 10.

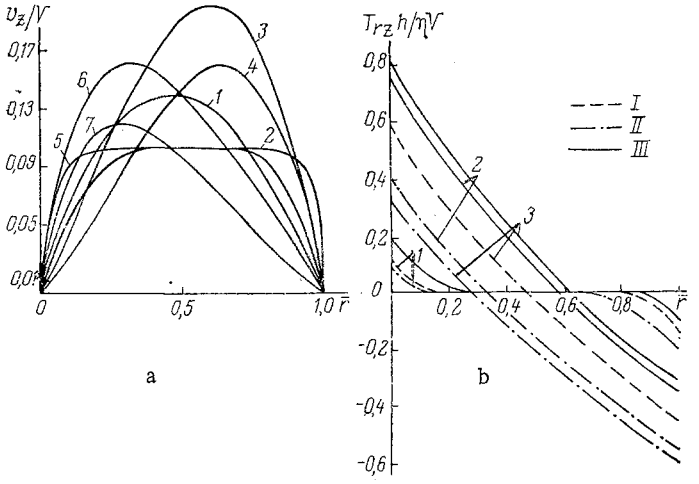

Fig. 3. Velocity (a) and tangent stress (b) profiles for non-isothermal liquid flow in the MBC model at El = 10, We = 1, $\alpha = 2$, δ = 0.1 at times: a) $t\eta/\rho h^2 = 0.1$ (2,5), 4.5 (3,6), 30 (1,4,7); b) $t\eta/\rho h^2 = 0.1$ (1), 4.5 (2), 30 (3), at: a) $\nu = 0$ (1), 0.25 (5,6,7), −0.25 (2,3,4); b) $\nu = 0$ (I), 0.25 (II), −0.25 (III).

NOTATION

r, Φ, z, cylindrical coordinates; v_z, velocity; T_{rz}, tangent stress; θ, temperature; $\partial p/\partial z$, pressure gradient; ρ, density; t, time; Q, flow process activation energy; R, universal gas constant; R_2, outer cylinder radius; R_1, inner cylinder radius.

LITERATURE CITED

1. J. Meissner, "The effect of temperature on the flow properties of low density polyethylene melt," Fourth International Congress on Rheology, Pt. III, Interscience, New York (1963).
2. J. S. Chong and D. M. Vezzi, "Unsteady flow of viscoelastic liquids," J. Appl. Polym. Sci., 14, No. 1, 17-34 (1970).
3. J. S. Chong and R. G. E. Franks, "Time-dependent flow of quasilinear viscoelastic fluids," J. Appl. Polym. Sci., 14, No. 6, 1639-1650 (1970).
4. I. Etter and W. R. Schowalter, "Unsteady flow of an Oldroyd fluid in a circular tube," Trans. Soc. Rheol., 9, No. 2, 351-369 (1965).
5. P. Townsend, "Numerical solutions of some unsteady flows of elasticoviscous liquids," Rheol. Acta, 12, No. 1, 13-18 (1973).
6. G. Akay, "Numerical solutions of some unsteady laminar flows of viscoelastic fluids in concentric annuli with axially moving boundaries," Rheol. Acta, 16, No. 2, 589-603 (1977).
7. M. F. Edwards, D. A. Nellist, and W. L. Wilkinson, "Unsteady laminar flow of non-Newtonian fluids in pipes," Chem. Eng. Sci., 27, No. 2, 295-306 (1972).
8. R. T. Balmer and M. A. Fiorina, "Unsteady flow of an inelastic power-law fluid in circular tube," J. Non-Newt. Fluid Mech., 7, No. 2/3, 189-198 (1980).
9. Z. P. Shul'man, S. M. Aleinikov, B. M. Khusid, and É. É. Yakobson, Rheological Equations of State of Flowing Polymer Media (Analysis of the State of the Art) [in Russian], Minsk (1981).
10. Z. P. Shul'man and B. M. Khusid, "The equivalence of the relaxation and integral models of polymer fluids," Dokl. Akad. Nauk SSSR, 262, No. 5, 1198-1201 (1982).
11. G. V. Vinogradov and A. Ya. Malkin, Polymer Rheology [in Russian], Khimiya, Moscow (1977).
12. B. S. Petukhov, Heat Exchange and Resistance in Laminar Liquid Flow in Tubes [in Russian], Nauka, Moscow (1967).